Computer analysis of the motion of a pendulum (3B pendulum apparatus)

Objectives

In this experiment you will:

- Use a data acquisition system to record the motion of a pendulum

- Make a mathematical model of the pendulum with the help of a data fitting application

- Find out how close the model comes to describing the real pendulum, and investigate the reasons for any deviations.

The data acquisition software and hardware

The STRIPCHART software (in the SCIENCE menu) is inspired by stripchart recorders, which use an electrically-activated pen to draw a trace on a moving strip of paper. The computer version has the advantage that you get the data in numerical form for later analysis. It takes voltage readings from channel 0 (and optionally channel 1) of the USB1208FS data acquisition module, which we will refer to as the ADC (analogue to digital converter).

The pendulum hardware

The low-friction bearing of this pendulum is simply two steel needles that rest in depressions in the supporting bar. To keep friction low, the angle of the pendulum is measured magnetically. Identify the two ceramic magnets in the bearing mechanism. They act on a magnetic field sensor inside the support rod, which measures a certain component of the field, using the Hall effect. Given that the field is directed from one magnet to the other, which component of the field must the Hall sensor measure if it is to give an output that is approximately linear in angle?   **Q**

Connect the output of this angle transducer to the the high and low channel 0 input terminals of the ADC terminal box. Run STRIPCHART. Adjust the settings of the

program, and the pendulum, so you can make a clear recording of the swinging of the pendulum and the decay in its amplitude. The pendulum is fitted with three polystyrene balls to increase the air resistance, so that decay occurs in a reasonable time.

Collecting preliminary data

Take some data to show the period and damping of the pendulum, and stick representative plots in your lab book. You should choose the voltage range and time between measurements to collect suitable data. Check that the data are approximately right by timing the pendulum using a stopwatch, and by calculating an approximate period T, assuming the simple pendulum formula, $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the pendulum length and g is the acceleration due to gravity.

Modelling the data

Acquire a clean data set showing the entire decay of the oscillations, with enough time resolution to show the detail of a single swing. You can zoom in on data after they are recorded by changing the time interval – this does not change the data, but does change the time axis. Save the data in a file, then start the QTIPLOT application and select *File | Import ASCII | Single File* to read in the saved data. Select the X and Y columns, then *Plot | Scatter* (or the plot-as-symbols toolbar icon) to produce a graph.

Most physicists would guess that the mathematical form of the decaying oscillation is

$$y = y_0 + A\exp\left(-\frac{t}{\tau}\right)\sin\left(\omega t + \phi\right)$$

where y is (angular) position, t is time, ω is the angular frequency, τ the decay time constant, and y_0 and ϕ are the initial conditions. What simplifying physical assumptions are implied by the sinusoidal and the exponential terms? **Q**

Qtiplot can try to fit this expression to the data, in other words it can adjust the parameters until the expression comes as close as possible to the data points (as judged by a least squares criterion). However, it helps to give it good initial guesses. Look carefully at your data, and write down your best estimates of y_0, A, τ, ω and ϕ. It may help to select the first few dozen points and make a separate plot of those.

Select *Analysis | Fit Wizard* and enter a user-defined function:
y0+A*exp(-x/tau)*sin(omega*x+phi). x stands for time t. Set the parameters to A, tau,

y0, omega, phi; these are the values that the fitting procedure will adjust. Select *Fit*, then enter your estimates of the parameters. Click the Fit button. After a few seconds, the best fit that the procedure found will be displayed. If it is completely crazy, either the function or the estimated parameters are bad. You can improve the appearance of the fitted curve by asking for a "uniform X function" with e.g. 1000 points in the *Custom Output* dialogue. When printing from Qtiplot, the page orientation **must be set to Portrait** (otherwise, printing fails). The size of the printed graph will be the same as the size on the screen.

How close does the exponentially decaying sine function come to your data? What does this tell you about the pendulum? **Q**

Extracting parameters

Even if the amplitude is not right, perhaps the frequency is fitted better. Use the fitted ω to determine the period of the pendulum, and hence the acceleration due to gravity, g. Note that you are not dealing with a simple pendulum (with the mass at the end), but one with the mass distributed along its length. The period of a pendulum made of a uniform one-dimensional rod of length l is

$$T = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}}.$$

A simple centre of mass argument might suggest that the factor should be 1/2 and not 2/3. What is wrong with the idea that the real pendulum is equivalent to a simple one with all its mass concentrated at the center of mass of the real pendulum? **Q**

Using the uncertainties of the fitted ω and a length measurement, compare your value for g with the accepted value, 9.81 ms^{-2}. Since you know what the result "should" be, try to determine the importance of random measuring errors, systematic errors (i.e. defects in the model), and other approximations you are making. **Q**

Refining the model

The expressions given above for the period of a pendulum assume that the angular amplitude of the swing, θ_0, is small. In fact, the frequency ω is expected to vary slightly with θ_0, as given by

$$\omega = \omega_0\left(1 - \frac{\theta_0^2}{16} + ...\right).$$

It therefore might be important to look at how the changing amplitude affects things. To evaluate θ_0, you will need to convert the peak transducer voltage (A above) to angle in radians.

Here are suggestions of how you might proceed, but you can try your own idea:

- Use the From/To options in the fit wizard to fit small regions of the data piece-wise. Look for systematic changes in ω. Plot a graph showing these changes.

- Alter the exponential term in the fitting function to get a better fit. Does this change the fitted ω? If you find a good function, what does this imply about the friction of the pendulum?

- Take some fresh data for different amplitudes

<div align="right">MSC 2015-10-21</div>