Computer analysis of the oscillations of a vertical beam

Objectives

In this experiment you will:

- Use a data acquisition system to record the oscillations of a beam

- Extract the best values for the period of oscillation by data fitting

- Understand the oscillations and eventual instability of a vertical beam in terms of a mathematical model.

The data acquisition software and hardware

The STRIPCHART software (in the SCIENCE menu) is inspired by stripchart recorders, which use an electrically-activated pen to draw a trace on a moving strip of paper. The computer version has the advantage that you get the data in numerical form for later analysis. It takes voltage readings from channel 0 (and optionally others) of the USB1208FS data acquisition module, which we will refer to as the ADC (analogue to digital converter).

Setting up the strain gauge bridge

The aluminium beam has strain gauges glued to it to measure its distortion. Please do not touch them, as they are fragile. A strain gauge bridge is used to provide a large output signal. You should find out the principles of how a strain gauge works. Connect the output of the bridge to the ADC input.   **Q**

Using an initial displacement of about 30 mm, set the beam vibrating, then run STRIPCHART. Adjust the its settings so you can make a clear recording of the oscillations (10 cycles on the screen is a reasonable place to start). Do not let the beam vibrate wildly, because it can be damaged by excessive bending.

Period as a function of mass

The main part of the experiment is to find out and understand how the period of the oscillating beam depends on the mass mounted at its vibrating upper end. As a first guess, you might suppose that the effective spring constant k of the beam, along with the mass m, would produce simple harmonic motion of period $T = 2\pi\sqrt{\dfrac{m}{k}}$.

Take some data to find the period of oscillations, as mass is added to the end of the beam – keep the load symmetric about the beam. You should choose the voltage range and time between measurements to collect suitable data. Do not exceed about 1 kg load.

A crude way to find the period from the data is to estimate the time for a whole number of cycles (e.g. from the screen); however, this throws away all the data points except those at the ends. A better way is to use a fitting procedure – not a straight line fit, but a sine wave fit.

Fitting the oscillatory data

Acquire a clean data set showing several oscillations, with enough time resolution to show the detail. Save the data in a file, then start the QTIPLOT application and select *File | Import ASCII* to read in the saved data. Select the X and Y columns, then *Plot | Scatter* to produce a graph.

A general expression for a sinusoidal oscillation is

$$y = y_0 + A\sin\left(2\pi\frac{x}{T} + \phi\right)$$

where y is displacement, x is time, T is the period, and y_0 and ϕ are the initial conditions.

Qtiplot can try to fit this expression to the data, in other words it can adjust the parameters until the expression comes as close as possible to all the data points (as judged by a least squares criterion). However, it is important to give it good initial guesses. Look carefully at your data, and make your best estimates of y_0, A, T and ϕ.

Select *Analysis | Fit Wizard* and enter a user-defined function:

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y0+A*sin(2*pi*x/T+phi).
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The parameters should be displayed as **A, T, phi, y0**. These are the values that the fitting procedure will adjust. Click the arrow to open the fitting session. Enter your estimates of the parameters; the Preview option can help you get good estimates. Click the Fit button. After a moment, the best fit that the procedure found will be displayed. If it is completely crazy, either the function or the estimated parameters are bad. You can improve the appearance of the fitted curve by asking for a "uniform X function" with e.g. 1000 points in the *Custom Output* dialogue.

How close does the sine function come to your data? What does this tell you about the oscillations? Do any systematic deviations matter in extracting the period?   **Q**

Use this method (or any other that you think is as good) to get values of period for a range of masses. Stick representative plots in your lab book. When printing from Qtiplot, the size of the printed graph will be the same as the size on the screen; selecting portrait orientation for the paper may be helpful.

Mass dependence

Produce a graph showing period as a function of mass. You can use Qtiplot or other methods to do this. Consider what to do about the mass of the screw arrangement that holds the masses to the bar, and the mass of the bar itself. Is your graph consistent **Q** with the square root dependence predicted above?

Comparison with a more realistic model

The spring and mass model ignores one important feature of the present geometry: as the bar bends, the mass falls slightly, so its gravitational potential energy decreases, at the same time as the strain potential energy in the bar increases. In figure 1, we approximate the path of the mass as a circular arc.
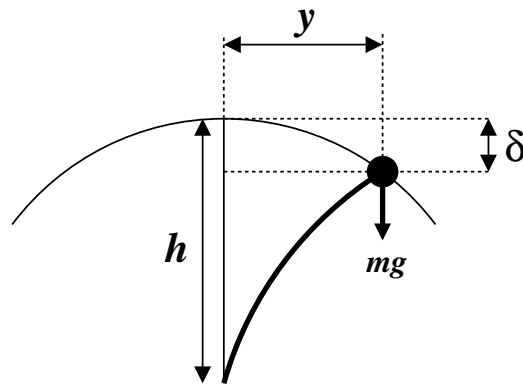
Figure 1: Approximate geometry of the bending beam

Show that in this case, the effective spring constant k' is given by **Q**

$$k' \approx k - \frac{mg}{h}.$$

Hint: The potential energy of the beam of spring constant k is $+\frac{1}{2}ky^2$. Combine this with the potential energy of the mass $-mg\delta$, and write the result in the form $\frac{1}{2}k'y^2$.

Thus we expect the period to be given by $T = 2\pi \sqrt{\dfrac{m}{k - \dfrac{mg}{h}}}$, where h is the active length

of the bar, and k the spring constant of the bar itself, $k = \dfrac{3EI}{h^3}$. E is Young's modulus for the material of the bar; I is the moment of inertia of the bar cross section, $I = \frac{1}{12}ab^3$; b is the beam thickness (in the direction of bending) and a is its width.

Measure the geometrical parameters, and using the value $E = 7\text{x}10^{10}\,\text{Nm}^{-2}$ for the aluminium alloy bar, plot a theoretical curve for T as a function of mass. You could use Freemat (Matlab clone) or other methods for this. Compare the theoretical curve with your data.

Instability

Clearly, the model predicts a divergence when $\dfrac{mg}{h} = k$, that is, when $m = \dfrac{3EI}{h^2g}$. What would this mean for the beam? Predict how much you can extend the provided weighted **Q** steel tape before it won't stay upright (E for steel is about $2\text{x}10^{11}\,\text{Nm}^{-2}$). Experiment to see how close the theory is. Why might it not match exactly?

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