Audio frequency spectrometer

Introduction

The SPECT software (in the SCIENCE menu) acquires voltage versus time data like an oscilloscope; but it also performs a Fourier analysis on the data, and displays its constituent frequencies as a spectrum.

In this experiment you will learn about the limitations of sampled data, the frequency domain data available from a Fast Fourier Transform spectrometer, and will use the spectrometer to perform analysis of the vibrational modes of a bar.

Comparing digitised data with the original signal

Using an oscilloscope, set the signal generator to provide a sine wave of a little less than 2 V peak-peak. Use a frequency range of 10 or 20 kHz, and set the frequency near the bottom of the range, at about 300 Hz. Use a BNC T-piece to connect channel 0 of the data acquisition system to the same signal. Run SPECT, taking 128 points in 20 ms.

Compare the time-domain signal shown by SPECT with that on the oscilloscope, as you slowly increase the frequency of the generator. You may find it easier to see the order in which the data points were recorded using the "join dots" function in SPECT. Notice also how the peak in the frequency domain window moves. Record your observations.

Notice that the digitised data can appear quite different to the real signal, as shown by the oscilloscope. Nevertheless, the peak in the frequency domain graph should track the actual frequency. Record what happens as you continue to increase the generator frequency beyond the upper limit of the frequency domain graph.

Digital data and the discrete Fourier transform

Any periodic signal $V(t)$ of period T can be represented by a Fourier series, i.e. a sum of the form

$$V(t) = \sum_{n=0}^{\infty} \left(a_n \cos \frac{2\pi n}{T} t + b_n \sin \frac{2\pi n}{T} t \right)$$

where the $n = 1$ term is known as the fundamental, and $n = 2, 3...$ are the second, third... harmonics. Calculation of the a and b coefficients will not be discussed here; almost all software, including SPECT, uses a method known as the Fast Fourier Transform (FFT). SPECT plots $\sqrt{a_n^2 + b_n^2}$ versus $\frac{n}{T}$ to make its frequency domain plot. What information is kept when a and b are combined in this way? What is being thrown away? What is the $n = 0$ term?

A real digitised signal has two limitations: the first is that the measurement time is finite. The FFT *assumes* that the signal repeats at the end of the measurement time, in other words T is set to the total measurement time. It follows that the frequencies that it can represent are $\frac{n}{T}$ for integer n. Verify that the spikes that can appear in the SPECT frequency display are consistent with its “Total time” setting. Try several settings, and record your findings. You may find the “Log” button helpful to make even very small spikes visible.

The second limitation is that the total number of samples (measured points) is finite. Given $N = 128$ data points, it is not possible to calculate more than 128 independent values of a_n and b_n. In fact, only a_0 and $\sqrt{a_n^2 + b_n^2}$ for $n = 1...64$ can be calculated (that sounds like one too many, but the $n = 0$ and $n = 64$ terms each contain only one coefficient). Verify that the total number of output frequencies depends on the “N points” setting in the correct way, recording your results.

The Nyquist frequency

Taking the limitations on frequency interval and number of output frequencies into account, it is obvious that there is a topmost frequency that can be represented. This is known as the Nyquist frequency, F_N. Write down an expression for F_N in terms of the time between successive measurements, $T_s = T/N$. What do you find happens if you apply a sine wave of frequency greater than F_N to SPECT?

Apply a sine wave of frequency $2F_N$ to SPECT. With careful adjustment, SPECT should see it as zero frequency (DC). Not only is the apparent frequency badly wrong, but the time domain plot agrees with it! Record your findings, contrasting the digitised data with what the analogue oscilloscope is telling you.

Draw a sketch of a few cycles of a periodic waveform, marked to show where it would be sampled if it were at the Nyquist frequency. How many samples are there per cycle? On another sketch, show why a signal of double the frequency would, if sampled at the same rate, appear to be a constant, and not oscillating at all.

The problem of the frequency ambiguity is known as aliasing, and is always a risk in sampled data systems. How could you guard against it?

Complex waveforms

Set the function generator to provide a square wave, and use SPECT to display its constituent sine wave components. You will need to apply what you have learnt about frequency resolution and the Nyquist frequency to get useful data. Hint: look for harmonics up to the 10th or so.

Do the same with whatever other waveforms your generator can provide: triangular, distorted sine (e.g. by turning up the DC offset or amplitude until clipping occurs), amplitude modulated... Be as quantitative as possible about what harmonics are present, and the systematic behaviour of their amplitudes. Stick suitable sketches or printouts in your lab book.

Recording sound

Connect a microphone via a GP1 microphone amplifier to the system in place of the signal generator. The microphone may need a battery inserting or swithcing on. Be sure to turn it off when you leave.

Adjust the controls to record the sound of your voice, or any musical (?) noise you can make. What range of frequencies are in your voice? Can you distinguish different vowel sounds?

Modal analysis

The metal bar provided will ring like a bell when struck. It has several modes of vibration, see figure 1. The modes have, in general, different frequencies. Various

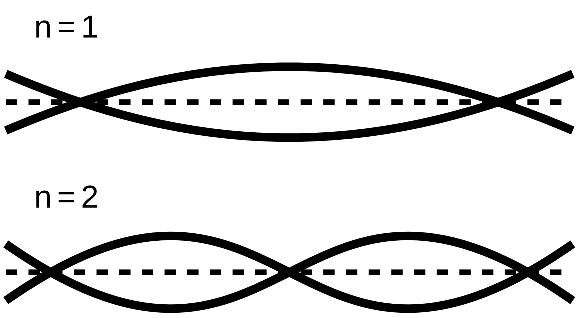

Figure 1: Some flexure modes of a bar

mixtures of these modes will be excited depending on how it is struck and how it is supported, and they will decay at different rates. You will be using the SPECT Fourier spectrometer to record the superposed oscillations, and extract their individual frequencies.

Record some rings of the bar, and measure some mode frequencies. The magnitudes can vary quite a lot; the “log” display may help. Measure the mode frequencies as accurately as you can.

Analysis

The flexural modes satisfy the equation

$$c^2k^2\frac{d^4u}{dx^4}+\frac{d^2u}{dt^2}=0,$$

in which $c=\sqrt{E/\rho}$, where E is Young’s modulus and ρ is the density; k is the radius of gyration ($k^2=I_C/A$, where I_C is the second moment of area about the central axis and A is the cross-sectional area). For a round bar, $k=d/4$ and for a square bar, $k=d/\sqrt{12}$, where d is the diameter or side.

Numbering the flexural modes $n=1,2,...$ the frequencies of the modes for a free bar of length L are given by the expression

$$f_n=\frac{m_n^2k}{L^2}\sqrt{\frac{E}{\rho}}$$

where m_n^2 are shown in the table.[1]

	$\frac{d}{L}=0$	$\frac{d}{L}=0.052$
m_1^2	3.561	3.538
m_2^2	9.815	9.683
m_3^2	19.234	18.701
m_4^2	31.807	30.574

the values for $d/L=0.052$ are for square bar; a solution for round bar is not available.

Determine whether the frequencies you have measured are consistent with these predictions, within the limits of your measurements. Note that there may be other modes

[1] M. Hussey, Fundamentals of Mechanical Vibrations, MacMillan (1983), pp.157-169; G.W. Van Senten, Introduction to Mechanical Vibration, Philips Technical Library (1961) pp. 29-32; W.T. Thomson, Journal of the Acoustical Society of America, **11**, 198 (1939).

not given by the formula! Consult any colleagues with different bars, to see whether the length dependence is as described.

Measure the parameters of your bar and calculate a value for Young's modulus. The density of aluminium is 2710 kgm^{-3}; of brass 8500 kgm^{-3}.

Suppose the bar were a manufactured part. How could you use its resonant modes to determine automatically whether it had a manufacturing defect?

MSC 2007-10-04