X-ray Powder Diffraction

Aims and Objectives

- Measure the lattice parameters of Si, NaCl, KCl and CsCl.
- Determine the structure factors for the above compounds.
- Use an x-ray generator and operate a two-circle powder diffractometer.

Introduction

Most solids have crystal structures, where the position of individual atoms is well known. To find out the structure of a given material, X-ray diffraction is the most commonly used experimental tool. X-rays are fired at a sample and interact with density fluctuations (i.e. the atomic positions), leading to reflections off atomic planes. By measuring these reflections, the structure can be determined. In this experiment, you will look at a variety of salts and determine their structures.

Background

A crystalline solid is an ordered state. If we know where one atom is, then we know where all of the other atoms in the material are. In practice, of course, atoms are constantly moving due to thermal and quantum fluctuations. This information is contained in two portions: the **lattice**, and the **basis**. The **lattice** consists of a series of lattice points that are symmetrically identical, i.e. if the lattice extended infinitely, it would be impossible to distinguish between the lattice points. These lattice points are commonly placed on atomic sites, although this is merely as a convenience. By means of a tiling system, the entire lattice can be constructed by designating a **unit cell**, which is defined by **lattice translation vectors**, and then tiling it across 3D space as necessary to represent the crystal in question. There may be more than one atom associated with each lattice point, or there may be more than one lattice point in the unit cell, in which case the position of the atoms in the unit cell may be described using **basis vectors**, which are fractional coordinates of the unit cell construction vectors.

How might atomic motion affect the discussion below?

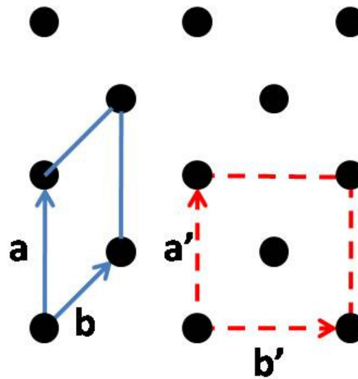

Figure 1: A sample two-dimensional lattice, illustrating possible unit cells and associated lattice translation vectors. For the dashed unit cell, the basis vectors for the lattice points are (0,0) and (1/2, 1/2).

Planes

Crystals that we observe on the macroscopic level often have facets, e.g. salt crystals form cubes. These faces correspond to particular crystal planes. If a crystal can be cleaved along a particular plane, an interface that is flat on an atomic scale can be formed. Knowing where planes are and the distances between them is very important for categorizing and identifying crystal structures.

Planes are labelled using **Miller indices**. Consider a plane as shown in Fig. 2 in red. Choose the unit cell and lattice translation vectors that you wish to use. Choose a particular lattice point and then find where the planes intercept along the lattice translation vectors. In this case, the planes intercept along the **b** axis at the next lattice point, and half-way to the lattice point in the **a** direction. The intercepts are therefore at $(\frac{\mathbf{a}}{2}, \frac{\mathbf{b}}{1})$. The Miller indices for these planes are (21). If the planes of interest lie parallel to a particular lattice translation vector, than the corresponding index in that case is 0.

In 3D, if the planes intercept the unit cell axes at

$$\left(\frac{\mathbf{a}}{h}, \frac{\mathbf{b}}{k}, \frac{\mathbf{c}}{l}\right) \tag{1}$$

then the Miller indices describing the plane are (hkl).

In high symmetry conditions, such as cubic lattices, several planes or directions may be symmetrically identical, e.g. the (100) plane looks the same as the (010) and (001) planes. To refer to all of these symmetrically equivalent planes, we use curly brackets: $\{100\}$.

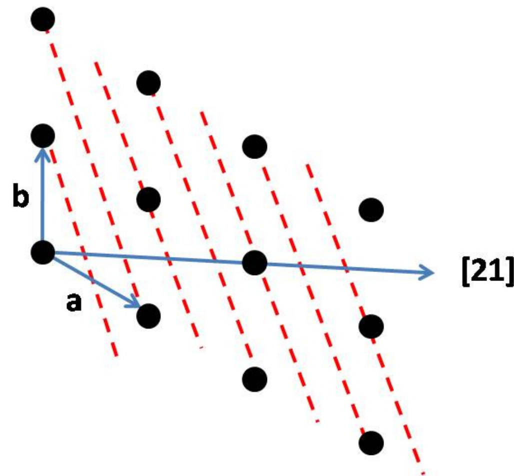

Figure 2: (21) planes.

Bragg's Law

Suppose we have a crystal of unknown origin that we wish to identify. We want to identify the unit cell, and so we need to know the size and direction of the lattice vectors, and the position of atoms in the basis, as well as the type of atoms. One way to get some of this information is via diffraction.

Consider the planes that we discussed earlier. They consist of uniform density variations in the material separated by a distance d. Suppose a plane wave of some radiation, such as light, is incident on these planes at some angle θ. Let us assume that the radiation will be specularly reflected by these planes of atoms.

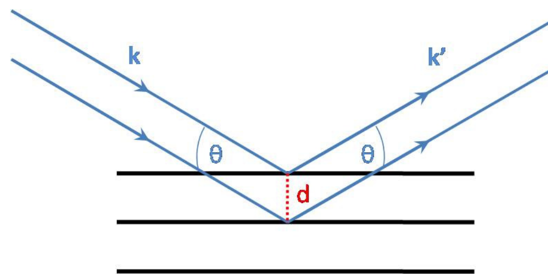

Figure 3: Reflection from planes of spacing d.

To get constructive interference at the detector from the two planes, the waves must be in phase. The extra path length from the plane below is $2d\sin\theta$, and so if

$$2d\sin\theta = n\lambda \tag{2}$$

where n is an integer and λ is the wavelength of the incident radiation, the outgoing beams will be in phase and give a strong reflected beam. If this condition is not met, there will be no constructive interference signal. This equation is **Bragg's law**.

This means that λ must be in the right range for probing atomic distances. A typical inter-atomic distance is ~ 3 Å. This means that x-rays are the right type of photon. If the wavelength is a fixed, known value, we can look for strong reflected signals at certain angles. By measuring the angle at which they appear, usually referred to as 2θ, we can calculate d and hence characterize the plane spacings. If we know several plane spacings, then with some detective work, we can extract the crystal structure.

For a cubic material, the plane spacings are given by

What is the general formula?

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}. \quad (3)$$

The Laue condition

Max von Laue developed a more rigorous derivation for the appearance of strong reflections from crystals that also introduces the useful concept of reciprocal space. You can find out about this method in the recommmended reading. This method makes no assumptions about planes, but merely relies on individual atoms scattering elastically. The final result is that the scattered x-ray amplitude varies as

$$\psi_{sc} \propto \sum_{l} \mathrm{e}^{-i\kappa\cdot\mathbf{r}_l} \sum_{m} b_m \,\mathrm{e}^{-i\kappa\cdot\mathbf{r}_m}. \quad (4)$$

where $\kappa = \mathbf{k}' - \mathbf{k}$ is the **scattering vector**, $\mathbf{r}_l$ is the position vector of a given unit cell and $\mathbf{r_m}$ is the position vector of atoms inside a unit cell. $\mathbf{r}_l$ is written in terms of the lattice translation vectors and κ is written in terms of reciprocal lattice vectors, which are defined as:

$$\mathbf{a}^* = 2\pi\frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}]} \;;\; \mathbf{b}^* = 2\pi\frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}]} \;;\; \mathbf{c}^* = 2\pi\frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}]}. \quad (5)$$

These vectors are the reciprocals of the unit cell axes, and are inverse lengths. The space defined by them may be referred to as **reciprocal space**.

When the scattering vector has the following value

$$\mathbf{G}_{hkl} = h\mathbf{a}^* + k\mathbf{b}^* + l\mathbf{c}^* \quad (6)$$

it matches a reciprocal lattice vector, and will give rise to a strong reflected signal. Diffraction occurs when the scattering vector κ is a reciprocal lattice vector $\mathbf{G}_{hkl}$, where (hkl) are the Miller indices of the plane being reflected off.

Can you confirm that this is true?

Structure factor

Having dealt with the lattice, we now turn to the effects of the contents of the unit cell.

$$F = \sum_m b_m \mathrm{e}^{-i\mathbf{K}\cdot\mathbf{r}_m} \tag{7}$$

This is commonly referred to as the **structure factor**. In a unit cell containing only one atom, this term reduces to $F = b_m$, since $\mathbf{r}_m = 0\mathbf{a} + 0\mathbf{b} + 0\mathbf{c}$. However, many materials have more than one atom per unit cell. Silicon has a face-centered cubic structure, with atoms at (0,0,0), (1/2,1/2,0), (1/2,0,1/2) and (0,1/2,1/2) inside the unit cell. What effect does this have on the structure factor for planes of the type 100, 200, 110, 111 etc.? Can you determine a general rule?

Experimental Method

The x-rays generated in this experiment have an energy of $\sim$ 8 keV. Such x-rays are readily absorbed by bodily tissue and can cause severe burns if sensible precautions are not taken. There are several safety measures and hardware interlocks that must be in place before you can turn on the x-ray set and open the x-ray shutter.

X-Ray Generator

To turn the x-ray set on, you will need a key (attached to a red fob) to release the emergency stop button on the wall behind the x-ray generator. You can obtain this from one of the demonstrators. Check that all of the perspex windows are closed before turning the generator on.

- The red button should be pulled out to supply power to the generator, and the mains switch should be on.
- Turn the water on, using the tap on the back wall near the floor. If there is no water flowing, the x-ray generator will not turn on.
- Turn the generator on by pressing the GENERATOR I button on the front panel. The 'X-Rays On' lights should come on and you should hear a clunk.
- Slowly raise the kV to 40 kV, using the dial on the front.
- Slowly raise the mA to 30 mA, using the dial on the front.

- To open a shutter, press simultaneously the SHUTTERS button and and the I button underneath the 2. This opens the shutter to the 2-circle diffractometer. A light should appear below this shutter, and you should hear a loud click. If you do not hear it, the shutter is not open. To close the shutter, press SHUTTERS AND 2 0.

If you turn the water off or hit the mains emergency stop button, the generator will turn off. There is an interlock system: if you open one of the perspex windows when the shutter is open, the generator will turn off. When changing sample, if the cover is not placed correctly, the shutter will not open.

Notice that the current setting (in mA) never exceeds the voltage (in kV). Maintain this relationship when you turn the machine off at the end of the day, as follows:

- Reduce the current slowly to zero
- Reduce the voltage to zero
- Press GENERATOR 0
- Leave the water running for several minutes after turning off the set to remove excess heat from the x-ray tube.

The x-ray tube has a copper anode. The characteristic wavelengths produced by this tube are:

Cu K_{α_1}	1.54056 Å
Cu K_{α_2}	1.54439 Å
Cu K_{β}	1.39222 Å

The K_{β} x-rays can be filtered out using a nickel filter. A range of filters (including no filter) are available, and are located on a wheel close to the central x-ray pillar. Check which filter is in place before running a scan.

Two-circle diffractometer

The two-circle diffractometer is controlled using the computer and control box provided. Ask a demonstrator for a user name. The control program is called "xrd", and should be found in the favourites or science menu. Turn on the control box before starting xrd. Once you open this program, the diffractometer will start to move if mains power is on. This is the calibration of the angular position of the detector. Once the detector arm has stopped moving, you may begin to control the diffractometer.

A scan is set up by specifying the start and end angles, the angular velocity and the count time desired. The graph on screen will show all data collected until the 'Clear Data' button is pushed. The data may be saved in a tab separated text file, for use elsewhere. The angular range you can cover is from 10° to 140°. If the angle is close to 0° an interlock is activated and the shutter will close.

Sample preparation

The first sample to look at is a compacted silicon powder pellet. Ask a demonstrator to show you how to load a sample. Other powder samples must be loaded into aluminium plates. The powder should be packed down and then a strip of Magic™tape placed across it to stop the powder from spilling out. You may find it useful to use one piece of tape to pick up spilled bits of powder from the plate, and then remove this and replace with a fresh piece of tape. Please check that no powder will escape before installing a new sample. Clean out the chamber of this happens. Remember to label your samples.

Analysis

The data are saved in a tab delimited text file. The program fityk allows you to load multiple data files, and to carry out peak fitting to obtain information about the position, width and integrated intensity of the diffraction peaks observed in the experiment. You can alternatively use the data file with software of you choice.

Suggested investigations

Start by taking a scan of the silicon pellet to investigate suitable scan rates. Start with 20-100°, at 0.2°/s, with 1s counts. You should be able to see peaks within five minutes, but the data will be of poor quality. A reliable, detailed scan may take an hour to collect over the full range. Calculate the lattice parameter of silicon, and compare this with the standard literature value. You could also do a high resolution scan over just one peak, and try to explain its shape. Once you are confident in operating the diffractometer, you may turn to the main part of this investigation, a collection of salts.

Can you explain the origin of all of the peaks observed?

Prepare a sample using some of the table salt provided. This consists of small crystalline grains. After measuring this, using a pestle and mortar, grind some of these crystals into a powder, and compare the diffractograms obtained from the two samples. Explain the diffractograms and any differences that you observe.

Other samples are available, including are potassium chloride and low-sodium table salt. What can you say about these materials?

What structure do they have?

The general procedure for finding the lattice constant a from a powder diffraction pattern is to use equation 2 to get the plane spacings, then use equation 3 (in the cubic case) to determine the Miller indices h, k and l, using the fact that these must be small integers. Equation 3 should then give a consistent value of a for all peaks. It is helpful to work with the squares of the equations. See, for example, Warren (1990) section 5.3.

Recommended Reading

S. A. Holgate, *Understanding Solid State Physics*, pp 125-139, CRC Press (2010).

J.-A. Nielsen and D. McMorrow, *Elements of Modern X-Ray Physics*, Wiley (2001).

W. L. Bragg, *The Structure of Some Crystals as Indicated by their Diffraction of X-Rays*, Proc. Roy. Soc. **89**, 248 (1914).

B.E. Warren, *X-Ray Diffraction*, Dover 1990.

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