

Resonances of circuits and loudspeaker

Aims of this experiment

- Learn about resonance effects in AC circuits
- Investigate the resonances of a loudspeaker

Lab notebook

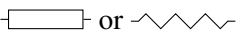
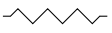


You should keep a written lab notebook while doing this and other experiments. It should contain

- Diagrams of circuits you simulated or built, or were built for you
- Waveforms, graphs, analysis, observations and explanations as prompted by this manual, and things that you decided to pursue. Be quantitative.
- Enough detail to enable someone to quickly pick up the thread of what you did and observed.

1 Reminder of AC circuits and Impedance

In DC circuits, the ratio of voltage V to current I , $\frac{V}{I}$ is known as the resistance, R , which is a real number of ohms. However, in AC circuits $\frac{V}{I}$ is known as the impedance Z ; the units are still ohms but it can be a complex quantity, allowing it to represent a phase angle as well as a magnitude. In this introduction, we shall only consider three electrical components as shown in table 1. You will note that the impedances

Table 1: Electrical components and their properties

Name	symbol	Properties	Impedance
Resistor, R	 or 	impedance constant with frequency	$Z = R$
Capacitor, C		impedance decreases with frequency	$Z = 1/j\omega C$
Inductor, L		impedance increases with frequency	$Z = j\omega L$

of capacitors and inductors are imaginary [in electronics, $j = \sqrt{-1}$, to avoid confusion with a current i].

If we represent a sinusoidal applied voltage $V(t) = V_0 \cos(\omega t)$ as $V_0 e^{j\omega t}$, then the current it produces in a capacitor is $V_0 e^{j\omega t} j\omega C = V_0 \omega C \exp[j(\omega t + \frac{\pi}{2})]$, which has a phase that is 90° ahead of the voltage.¹

2 Capacitor circuit

Use QucsStudio to make the capacitor circuit shown in Fig. 1 (For help with QucsStudio, see the introduction video in the **QucsStudio (circuit simulation)** software Canvas page, and **Video-1 LC-series-circuit.mp4** and **Video-2 saving-data-QucsStudio.mp4** provided with this experiment.)

¹ $e^{j\theta} = \cos \theta + j \sin \theta$, so with $\theta = \pi/2$, $j = e^{j\pi/2}$

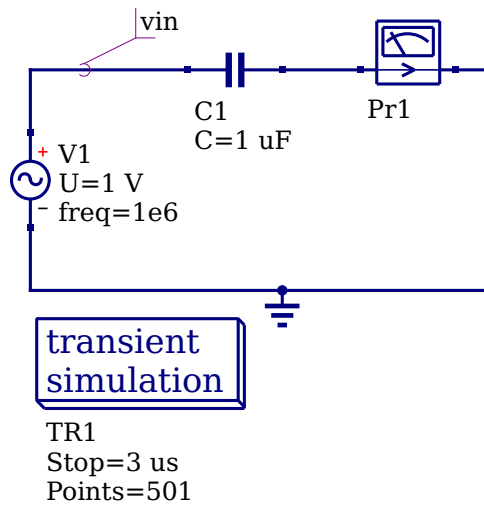


Figure 1: Circuit diagram

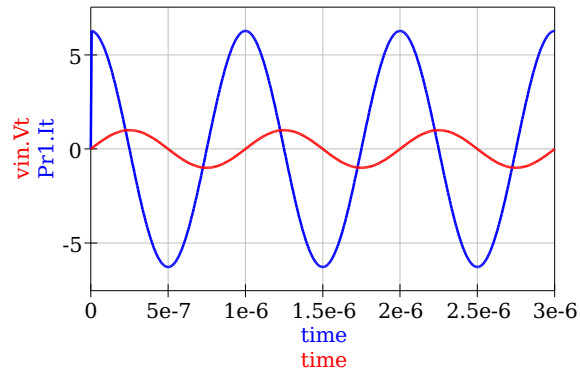


Figure 2: Voltage and current vs. time

Set the voltage source frequency to $1e6$ (1 MHz), the capacitance to $1 \mu\text{F}$ (μ is just **u** in QucsStudio), and the simulation time to $3 \mu\text{s}$. Plot the voltage and current as shown in Fig. 2. Are the phase difference and maximum value of the current what you would expect (remember $\omega = 2\pi f$)?

3 Resonant Circuits

3.1 L-C resonant circuit

Use QucsStudio to construct the LC (inductor-capacitor) circuit shown in Fig. 3.

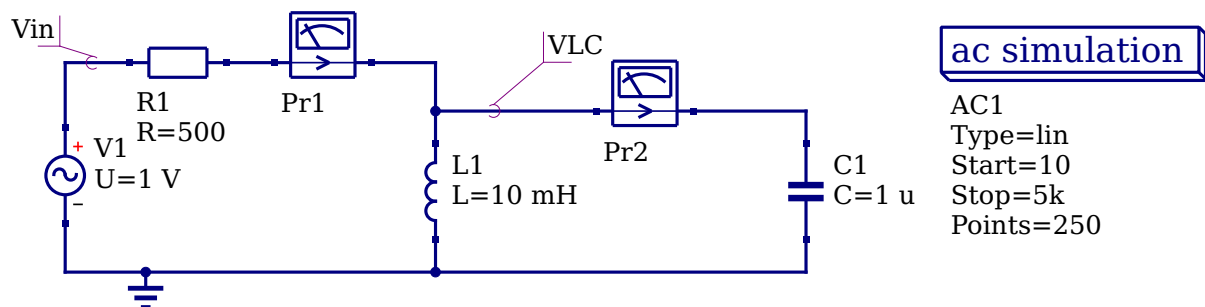


Figure 3: LC circuit

Run the simulation to plot the voltage across the parallel-connected LC part (**VLC.v** on the QUCS plot) and also plot its phase difference compared to the input, (**phase(VLC.v)** in QUCS) – see Video 1 for plotting the phase in a slightly different circuit. The current probes Pr1 and Pr2 don't affect the operation of the circuit.

- The peak of the resonant response is supposed to occur at the resonant frequency ω_0 , where the

magnitude of the impedances of the inductor and capacitor are equal. Investigate whether this is the case.

- In this circuit, the value of VLC at its peak should equal the applied voltage V_{in} . This implies (by Ohm's law in R1) that no current flows through R1 into the parallel combination of L1 and C1. How can you reconcile this with the fact that the impedance of neither L1 nor C1 is infinite? Hint: calculate the impedance of the parallel combination of L1 and C1 at ω_0 , taking into account their phase shifts by including the j factors.
- Add a second cartesian diagram (components » diagrams) to investigate the current measured by Pr1 and Pr2 (**Pr1.i** and **Pr2.i**). Is the current in R1 zero at resonance as previously stated? How does the current flowing in the inductor-capacitor loop respond?
- The maximum factor by which the circulating current measured by Pr2 at ω_0 exceeds the value of input current (Pr1) well away from resonance is known as the quality, or Q-factor, Q . In this circuit $Q = R_1 \sqrt{\frac{C_1}{L_1}}$ (in the limit of large Q). Compare your results with this expression. What happens if you increase R1 by a factor of 10?
- A second way of measuring Q is the ratio of the resonant frequency to the width of the resonance curve, measured at a height of $\frac{1}{\sqrt{2}}$ of its peak. Calculate this from your plot of VLC. You can use the "set marker on graph" button (plus sign on toolbar) to read values from the plot, and can drag the selected point around.
- Can you think of a useful application of a resonant circuit, especially if the capacitance and/or inductance are variable?

3.2 Loudspeaker circuit

Use QucsStudio to construct the simple speaker circuit shown in Fig. 4 (the loudspeaker can be found by clicking on the *Library* left hand tab and then in *Component Selection* select *diverse*).

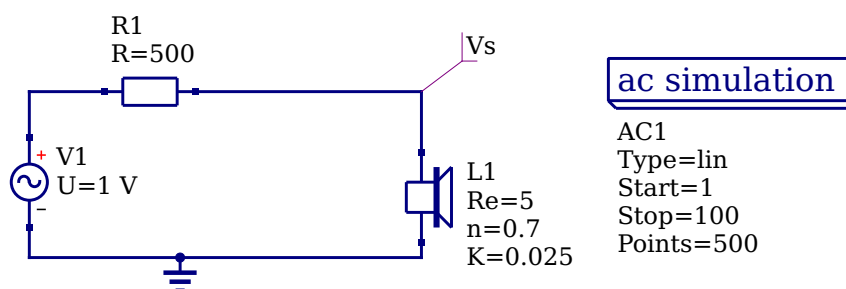


Figure 4: Speaker circuit

A typical speaker exhibits at least one resonance, from its mechanical construction. Even though it is not electronic in origin, QUCS can still model it, and we normally characterise such resonances the same way we do electronic ones.

- Run the simulation from 1 Hz to 100 Hz and characterise the resonance in terms of its resonant frequency and Q-factor (using the width method from section 3.1).

- Put a $20\ \Omega$ and then a $5\ \Omega$ resistor in parallel with the speaker. How does this affect the response of the speaker? Would this be a good or bad thing for the audio fidelity of the speaker?

4 Resonance of a real loudspeaker

In this part you will use analogue techniques to investigate the resonances of a loudspeaker, find out what causes them, and see how they can be eliminated.

4.1 Measuring the resonance

Watch the Video of the loudspeaker **Video-3 Loudspeaker.mp4**.

You will see the speaker is connected to a signal generator via a $100\ \Omega$ resistor. The input signal is also connected to channel 1 of an oscilloscope and the loudspeaker is connected to channel 2, as shown in figure 5.

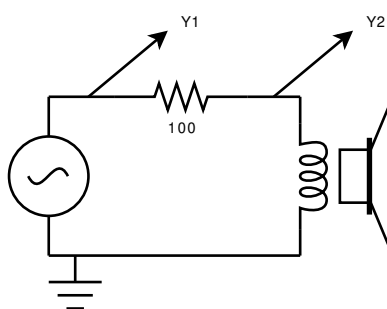


Figure 5: Connection of loudspeaker

Using a sine wave, the generator frequency was swept and the driving waveform (Y1) and the response waveform (Y2) on the oscilloscope can be observed (in the video). You will probably see a strong peak in the response at a low audio frequency. This resonance indicates that the speaker, and this way of driving it, are not of high quality.

Clearly, Y1 is the applied signal. The peak that you see in Y2 coincides with the frequency at which the sound is loud, and it is in fact closely connected with the motion of the loudspeaker cone. Find out how a loudspeaker works, and hence explain why motion of the cone produces voltage at Y2. An understanding of how the loudspeaker works will also make clear why Y1 is proportional to the force that moves the loudspeaker cone. Write your observations and explanations in your lab book.

Use the data on Y2 as a function of frequency (the "First Resonance" data in **speaker-data.xlsx**) to plot and fit the resonance curve. A reasonable function to fit would be a Lorentzian; this is in the SciDavis Fit Wizard under Built-in functions, where it is given as $y_0 + 2 * A / \pi * w / (4 * (x - x_c)^2 + w^2)$. It is important to enter good initial estimates of the parameters. The resonant frequency x_c is easy, as are y_0 , the y-value well away from resonance, and w , the full width at half height. The parameter A is more tricky. You can show that it should be $\pi/2$ times w times the height of the peak above y_0 . Use your fit to characterise the resonance as in section 3.1. x_c gives the resonant frequency, but for the Q you need the full width at $\sqrt{2}$ height.

Note from the video how the phase relationship between Y1 and Y2 change as you tune through the resonant peak; there is no need to measure it in detail.

4.2 Lissajous figures

It is easier to locate resonances by looking at the phase relationship between force and response rather than the ratio of amplitudes. At resonance, the phase difference changes rapidly, in fact in the present system it goes through zero. When the oscilloscope is switched to X/Y mode the pattern it draws (Y2 vs. Y1) is known as a Lissajous figure, and in this case should be an ellipse. Why is this?

Data from the oscilloscope are available at three different frequencies (see files **speaker-68Hz.txt** etc.). Import these into SciDAVis and plot the traces for Y1 and Y2. If using the File » Import ASCII function, set the separator to TAB before selecting the file.

If you select the one X and two Y columns, you will see the normal voltage vs. time display. But if you label the first Y column as X data (right click on heading » set column as » X), then select just this and the other Y column for plotting, you will get the Y1 vs. Y2 display that shows a Lissajous figure. At resonance, the ellipse collapses into a diagonal line (why?). By watching for this effect, further resonances can be located easily.

4.3 Identifying the resonances

Even when we know the resonant frequencies, we may not know what is resonating: where is the effective mass, and where is the effective spring? Your virtual lab partner has added one or more pea-sized pieces of Blotak to the centre of the loudspeaker cone (to increase its mass). He has then recorded the first resonance frequency vs. added mass (see data in **speaker-data.xlsx**). Plot and fit the data, taking into account the errors. By studying the final plot and χ^2 per degree of freedom, comment on whether the errors have been over or under estimated or are about right.

In the side of the loudspeaker, there is a hole that is normally blocked by a bung. Data around the first resonance has been taken with the hole unblocked (see **speaker-data.xlsx**). Fit these data and report the effect, if any, of unblocking the hole.

What do your results tell you about the origin of the resonances?

4.4 Damping the resonances

As configured here, the loudspeaker would not be very useful in a sound system because its resonances give it an uneven frequency response. Following the method of Section 3.2, your assistant has added a damping resistor in parallel with the speaker and re-measured the main resonance at Y2 (see file **resonance-with-parallel-resistor.xlsx**). Examine and comment on the effect on the main resonance of a 5 Ω resistor.

4.5 Transient response

Watch the video: **Video-4 Transient response.mp4**

Look at the response of the loudspeaker to a square wave at about 5 to 10 Hz. What kind of oscillations do

you see, and why? Investigate the effect of a $10\ \Omega$ resistor in parallel to the speaker (see the **transient*.txt** files). In fact, tests like this can reveal a lot about any signal processing system.