DEMO NOTES: *This manual contains demonstrator notes in blue italics*

Resonances of circuits and loudspeaker

Aims of this experiment

- Learn about resonance effects in AC circuits
- Investigate the resonances of a loudspeaker

Lab notebook

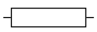
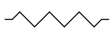
You should keep a written lab notebook while doing this and other experiments. It should contain

- Diagrams of circuits you simulated or built, or were built for you
- Waveforms, graphs, analysis, observations and explanations as prompted by this manual, and things that you decided to pursue. Be quantitative.
- Enough detail to enable someone to quickly pick up the thread of what you did and observed.

1 Reminder of AC circuits and Impedance

In DC circuits, the ratio of voltage V to current I, $\frac{V}{I}$ is known as the resistance, R, which a real number of ohms. However, in AC circuits $\frac{V}{I}$ is known as the impedance Z; the units are still ohms but it can be a complex quantity, allowing it to represent a phase angle as well as a magnitude. In this introduction, we shall only consider three electrical components as shown in table 1. You will note that the impedances of capacitors and inductors are imaginary [in electronics, $j = \sqrt{-1}$, to avoid confusion with a current i].

Table 1: Electrical components and their properties

Name	symbol	Properties	Impedance
Resistor, R	 or 	impedance constant with frequency	Z = R
Capacitor, C		impedance decreases with frequency	$Z = 1/j\omega C$
Inductor, L		impedance increases with frequency	$Z = j\omega L$

If we represent a sinusoidal applied voltage $V(t) = V_0 \cos(\omega t)$ as $V_0 e^{j\omega t}$, then the current it produces in a capacitor is $V_0\, e^{j\omega t}\, j\omega C = V_0\, \omega C \exp[j(\omega t + \frac{\pi}{2})]$, which has a phase that is 90° ahead of the voltage.[1]

2 Capacitor circuit

Use QucsStudio to make the capacitor circuit shown in Fig. 1 (For help with QucsStudio, see the introduction video in the **QucStudio (circuit simulation)** software Canvas page, and **Video-1 LC-series-circuit.mp4** and **Video-2 saving-data-QucsStudio.mp4** provided with this experiment.)

[1] $e^{j\theta} = \cos\theta + j\sin\theta$, so with $\theta = \pi/2$, $j = e^{j\pi/2}$

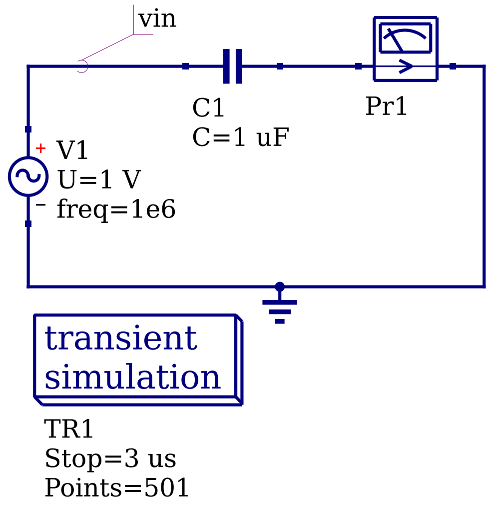


Figure 1: Circuit diagram

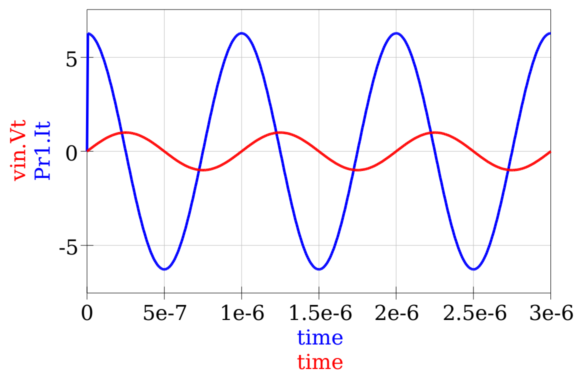


Figure 2: Voltage and current vs. time

Set the voltage source frequency to 1e6 (1 MHz), the capacitance to 1 μF (μ is just **u** in QucsStudio), and the simulation time to 3 μs. Plot the voltage and current as shown in Fig. 2. Are the phase difference and maximum value of the current what you would expect (remember $\omega = 2\pi f$)? DEMO NOTES: *They should be. Features in the red voltage trace occur at slightly later (larger) times than in the blue current trace, so the voltage is lagging the current, which is correct for a capacitor. By inspection, the amount of lag is $\frac{1}{4}$ cycle, 90°, or $\pi/2$.*

3 Resonant Circuits

3.1 L-C resonant circuit

Use QucsStudio to construct the LC (inductor-capacitor) circuit shown in Fig. 3.

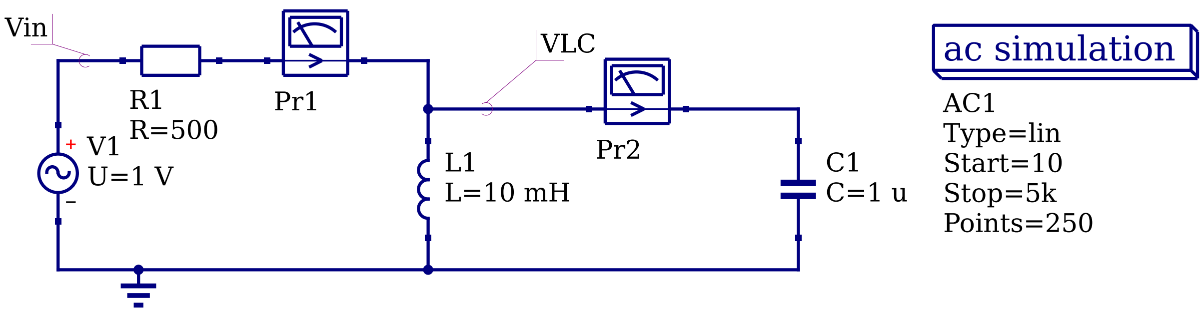


Figure 3: LC circuit

Run the simulation to plot the voltage across the parallel-connected LC part (`VLC.v` on the QUCS plot) and also plot its phase difference compared to the input, (`phase(VLC.v)` in QUCS) – see Video 1 for plotting the phase in a slightly different circuit. The current probes Pr1 and Pr2 don't affect the operation of the circuit.

- The peak of the resonant response is supposed to occur at the resonant frequency ω_0, where the magnitude of the impedances of the inductor and capacitor are equal. Investigate whether this is the case. DEMO NOTES: *Equating the inductor and capacitor impedances (without the j factors),* $\omega_0 = \frac{1}{\sqrt{LC}} = 10^4\ \text{rad}\cdot\text{s}^{-1} = 1591\ \text{Hz}$*, which is where the peak in* V_{LC} *should be on the graph.*
- In this circuit, the value of VLC at its peak should equal the applied voltage Vin. This implies (by Ohms's law in R1) that no current flows through R1 into the parallel combination of L1 and C1. How can you reconcile this with the fact that the impedance of neither L1 nor C1 is infinite? Hint: calculate the impedance of the parallel combination of L1 and C1 at ω_0, taking into account their phase shifts by including the j factors. DEMO NOTES: *Combining the impedances in parallel, this time keeping them complex,* $Z = \frac{j\omega L \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$*. This diverges at the resonant frequency* $\omega = \frac{1}{\sqrt{LC}}$*, giving an infinite net impedance at this frequency. There is current in the inductor, but it is cancelled out by a current of the same magnitude but opposite phase in the capacitor.*
- Add a second cartesian diagram (components » diagrams) to investigate the current measured by Pr1 and Pr2 (**`Pr1.i`** and **`Pr2.i`**). Is the current in R1 zero at resonance as previously stated? How does the current flowing in the inductor-capacitor loop respond? DEMO NOTES: *At* ω_0*, the current Pr1 goes to zero, but Pr2, the current circulating between the inductor and capacitor goes through a maximum.*
- The maximum factor by which the circulating current measured by Pr2 at ω_0 exceeds the value of input current (Pr1) well away from resonance is known as the quality, or Q-factor, Q. In this circuit $Q = R_1\sqrt{\frac{C_1}{L_1}}$ (in the limit of large Q). Compare your results with this expression. What happens if you increase R1 by a factor of 10? DEMO NOTES: *The formula gives* $Q = 5$ *with* $R1 = 500\Omega$*. The graph should show that the ratio between the peak of PR2 and the off-resonance Pr1 is of this size.*
- A second way of measuring Q is the ratio of the resonant frequency to the width of the resonance curve, measured at a height of $\frac{1}{\sqrt{2}}$ of its peak. Calculate this from your plot of VLC. You can use the "set marker on graph" button (plus sign on toolbar) to read values from the plot, and can drag the selected point around. DEMO NOTES: *It helps to plot e.g. 1000 points to be sure of one where you want to measure. I got* $f_0 = 1590\ \text{Hz} = 10000/2\pi\ \text{rad}\cdot\text{s}^{-1}$*. The peak is at 1 volt, so measuring at 0.707 volts, I got* $\Delta f = 1760 - 1440 = 320\ \text{Hz}$*, then 1590/320 = 4.97.*
- Can you think of a useful application of a resonant circuit, especially if the capacitance and/or inductance are variable? DEMO NOTES: *e.g. tuning a radio to pick out a station with the desired frequency (does anybody still do this?)*

3.2 Loudspeaker circuit

Use QucsStudio to construct the simple speaker circuit shown in Fig. 4 (the loudspeaker can be found by clicking on the *Library* left hand tab and then in *Component Selection* select *diverse*).

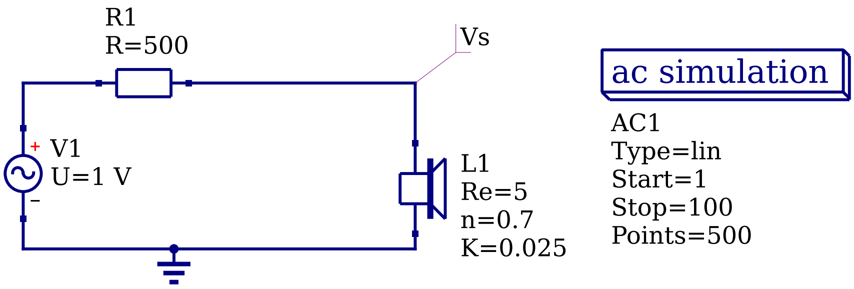


Figure 4: Speaker circuit

A typical speaker exhibits at least one resonance, from its mechanical construction. Even though it is not electronic in origin, QUCS can still model it, and we normally characterise such resonances the same way we do electronic ones.

- Run the simulation from 1 Hz to 100 Hz and characterise the resonance in terms of its resonant frequency and Q-factor (using the width method from section 3.1). DEMO NOTES: *Reading from the graph, I got* $f_0 = 34.8$ *Hz,* $\frac{1}{\sqrt{2}}$ *points at 28.6 and 42.1 Hz, giving* $Q = \frac{34.8}{42.1-28.6} = 2.58$.

- Put a 20 Ω and then a 5 Ω resistor in parallel with the speaker. How does this affect the response of the speaker? Would this be a good or bad thing for the audio fidelity of the speaker? DEMO NOTES: *With 20 Ω, the resonant frequency goes down a little to 34.5 Hz, and the Q decreases to 1.1. With 5 Ω, I get 33.3 Hz and Q = 0.43. In the latter case the resonance becomes quite asymmetric. A peak in the mechanical reponse of a speaker is a bad thing, because it would be louder for those frequencies than for others. Flattening it out by damping makes the effect less severe. It might be noted that the response around 30 Hz isn't very significant for ordinary music, but loud rumbling noises are still a bad thing. The observant might notice that as well as damping the resonance, the resistors form a potential divider that reduces the overall signal size a great deal. In practice, The 500 ohm resistor wouldn't be there; we have introduced it to exaggerate the effect.*

4 Resonance of a real loudspeaker

In this part you will use analogue techniques to investigate the resonances of a loudspeaker, find out what causes them, and see how they can be eliminated.

4.1 Measuring the resonance

Watch the Video of the loudspeaker **Video-3 Loudspeaker.mp4**.

You will see the speaker is connected to a signal generator via a 100 Ω resistor. The input signal is also connected to channel 1 of an oscilloscope and the loudspeaker is connected to channel 2, as shown in figure 5.

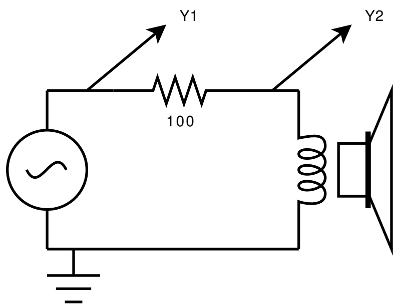


Figure 5: Connection of loudspeaker

Using a sine wave, the generator frequency was swept and the driving waveform (Y1) and the response waveform (Y2) on the oscilloscope can be observed (in the video). You will probably see a strong peak in the response at a low audio frequency. This resonance indicates that the speaker, and this way of driving it, are not of high quality.

Clearly, Y1 is the applied signal. The peak that you see in Y2 coincides with the frequency at which the sound is loud, and it is in fact closely connected with the motion of the loudspeaker cone. Find out how a loudspeaker works, and hence explain why motion of the cone produces voltage at Y2. An understanding of how the loudspeaker works will also make clear why Y1 is proportional to the force that moves the loudspeaker cone. Write your observations and explanations in your lab book. DEMO NOTES: *The speaker cone has a coil of wire attached to it, and the coil experiences a force in the field of a fixed permanent magnet. The force is proportional to the current in the coil, and so long as the 100 Ω resistor is the dominant resistance in the circuit, the current is proportional to the voltage at Y1. When the coil moves in the field, it generates a (back-)emf that can be seen at Y2. So long as this is significantly smaller than the voltage at Y1, the back-emf has little effect on the current.*

Use the data on Y2 as a function of frequency (the "First Resonance" data in **speaker-data.xlsx**) to plot and fit the resonance curve. A reasonable function to fit would be a Lorentzian; this is in the SciDavis Fit Wizard under Built-in functions, where it is given as $y0+2*A/PI*w/(4*(x-xc)^2+w^2)$. It is important to enter good initial estimates of the parameters. The resonant frequency xc is easy, as are $y0$, the y-value well away from resonance, and w, the full width at half height. The parameter A is more tricky. You can show that it should be $\pi/2$ times w times the height of the peak above $y0$. Use your fit to characterise the resonance as in section 3.1. xc gives the resonant frequency, but for the Q you need the full width at $\sqrt{2}$ height. DEMO NOTES: *Good starting values: w=40, A=60, Xc=120, y0=0.8, fitted; w=45.67, A=75.95, xc=119.5, y0=0.718. The width at at the $\sqrt{2}$ point is about 29, hence Q=119.5/29 = 4.1. Actually the Lorentzian as provided is appropriate to a power measurement rather than amplitude, but this is a bit too subtle, as is the fact that a Lorentzian describes the large Q case, which we are barely in. So let's allow some latitude on the Q measurement.*

Note from the video how the phase relationship between Y1 and Y2 change as you tune through the resonant peak; there is no need to measure it in detail.

4.2 Lissajous figures

It is easier to locate resonances by looking at the phase relationship between force and response rather than the ratio of amplitudes. At resonance, the phase difference changes rapidly, in fact in the present system it goes through zero. When the oscilloscope is switched to X/Y mode the pattern it draws (Y2 vs. Y1) is known as a Lissajous figure, and in this case should be an ellipse. Why is this?

Data from the oscilloscope are available at three different frequencies (see files **speaker-68Hz.txt** etc.). Import these into SciDAVies and plot the traces for Y1 and Y2. If using the File » Import ASCII function, set the separator to TAB before selecting the file.

If you select the one X and two Y columns, you will see the normal voltage vs. time display. But if you label the first Y column as X data (right click on heading » set column as » X), then select just this and the other Y column for plotting, you will get the Y1 vs. Y2 display that shows a Lissajous figure. At resonance, the ellipse collapses into a diagonal line (why?). By watching for this effect, further resonances can be located easily. DEMO NOTES: *There is no data file for the 120 Hz resonance, but one at 412 Hz is in the data provided. These Lissajous plots are parametric curves, where the parameter is time. A parametric plot of sine vs. cosine (90° phase difference) is a circle, while sine vs. sine (same phase) is a diagonal line. Intermediate phase differences show the circle collapsing through ellipses with diagonal axes to a diagonal line. It's easy to see a degree or two of phase shift this way.*

4.3 Identifying the resonances

Even when we know the resonant frequencies, we may not know what is resonating: where is the effective mass, and where is the effective spring? Your virtual lab partner has added one or more pea-sized pieces of Blutak to the centre of the loudspeaker cone (to increase its mass). He has then recorded the first resonance frequency vs. added mass (see data in **speaker-data.xlsx**). Plot and fit the data, taking into account the errors. By studying the final plot and χ^2 per degree of freedom, comment of whether the errors have been over or under estimated or are about right. DEMO NOTES: *The choice of fitting function is left to students. Linear looks fair, but is not a good choice. A/sqrt(m+x) is more physical (frequency inversely proportional to mass, with unknown starting mass m) and fits very well. Second order poly fits almost as well, but lacks physical insight.*

In the side of the loudspeaker, there is a hole that is normally blocked by a bung. Data around the first resonance has been taken with the hole unblocked (see **speaker-data.xlsx**). Fit these data and report the effect, if any, of unblocking the hole.

What do your results tell you about the origin of the resonances? DEMO NOTES: *The moving mass is clearly that of the speaker cone, with an estimated effective mass of 3.8g – adding to it reduces the resonant frequency. But the spring is not the air inside the cabinet, since unblocking it has negligible effect on the frequency (119.5 to 120.2 Hz). However, the damping of the resonance is increased a little by letting air flow in and out of the hole (width goes from 45 to 56 Hz)*

4.4 Damping the resonances

As configured here, the loudspeaker would not be very useful in a sound system because its resonances give it an uneven frequency response. Following the method of Section 3.2, your assistant has added a damping resistor in parallel with the speaker and re-measured the main resonance at Y2 (see file **resonance-with-parallel-resistor.xlsx**). Examine and comment on the effect on the main resonance of a 5 Ω resistor. DEMO NOTES: *The width of the resonance has roughly doubled to 89 Hz. The back emf generated by the motion of the coil is in such a direction as to drive a current that opposes the* *motion. The resistor allows this current to flow more easily. Dissipation in the resistor removes energy from the vibrating coil.*

4.5 Transient response

Watch the video: **Video-4 Transient response.mp4**
Look at the response of the loudspeaker to a square wave at about 5 to 10 Hz. What kind of oscillations do you see, and why? Investigate the effect of a 10 Ω resistor in parallel to the speaker (see the **transient*.txt** files). In fact, tests like this can reveal a lot about any signal processing system. DEMO NOTES: *The Fourier components in a square wave cover a range of frequencies (ideally an infinite series), so the speaker is effectively being tested with all of them at once. If they were all reproduced equally, the response would also be a square wave. Frequencies that are overrepresented in the output show up clearly as ringing on the square wave edges. The ringing at the resonant frequency is well shown by plotting the data, and is markedly reduced by the damping resistor (3 cycles to 1).*

DE 2020-10-10
MSC 2020-10-25