Precision wavelength measurement using a Fabry-Pérot etalon

Introduction

The main purpose of this experiment is to make a very precise measurement of the wavelength of a spectral line, which (we will assume) is initially known only approximately. This parallels the pioneering work of Fabry and Perot, in which they brought modern precision to the lines of the solar spectrum.

The etalon, devised by Fabry and Perot, is a kind of interferometer that attains very high resolution by combining multiple beams of light produced by the multiple reflections of a pair of parallel mirrors, see figure 1. The strategy of the experiment is to use some spectral lines of known wavelength to determine the spacing of the etalon mirrors, and then use that spacing to determine the unknown wavelength. An important part of performing the experiment is to understand how an etalon works, and produces its characteristic fringe pattern.

![Figure 1: Fabry-Perot etalon](image)

The constant deviation prism spectrometer

A prism spectrometer is used to provide a coarse separation of the spectral lines, in addition to the more precise etalon, see figure 2. You should first set up the spectrometer alone. Remove its covers, and remove the etalon, *taking care not to touch its optical surfaces or adjustment screws.*
Open the collimator slit fairly wide (~1mm) and illuminate it with the cadmium lamp. Adjust the wavelength selector until one or more spectral lines are visible through the eyepiece. Find out how to use the width, height and blackout controls of the slit. The eyepiece contains a pointer that can be locked in place. First focus the eyepiece (by moving it in and out) so that the pointer is in focus. Then adjust the telescope focus to get a sharp image of the slit. Close the slit so that the spectral lines do not overlap, and adjust the position of the rectangular aperture in the eyepiece so you can see a wide span of the spectrum. Using the wavelength selector, survey the spectrum of the cadmium lamp, and identify the lines shown in table 1.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Colour</th>
</tr>
</thead>
<tbody>
<tr>
<td>643.8470</td>
<td>red</td>
</tr>
<tr>
<td>508.5824</td>
<td>green</td>
</tr>
<tr>
<td>479.9914</td>
<td>cyan</td>
</tr>
<tr>
<td>467.8149</td>
<td>blue</td>
</tr>
<tr>
<td>441 approx.</td>
<td>violet</td>
</tr>
</tbody>
</table>

Table 1: Bright lines in the Cd spectrum

Take some wavelength readings from the wavelength selector drum, and comment on their accuracy. Observe how the wavelength selector adjusts the prism. Why is
this instrument known as a “constant deviation” spectrometer? How does the Pellin-Broca prism it contains differ from the equilateral prism you might find in simpler spectrometers - see figure 3?

![Figure 3: Pellin-Broca constant deviation prism, showing imaginary division into simpler prisms](image)

**The etalon**

Check that the etalon is correctly adjusted by looking through it at a patch of monochromatic light - a sodium lamp with some diffusion works well. With your eye focused at infinity, you should see a set of fringes in the form of concentric rings when looking exactly perpendicular to the mirrors. When you move your head from side to side, the rings should follow your eye, but should not appear to grow from or shrink towards their centre. If the fringes do not appear as described, consult a demonstrator. Some small change in ring size is to be expected, because the mirrors are not perfectly flat, and so cannot be perfectly parallel. Estimate the amount of non-parallelism, as a fraction of a fringe. What error in spacing does it correspond to?

Place the etalon in the spectrometer, carefully mating its feet with the kinematic mounting points. Each spectral line should be broken into segments by the action of the etalon: what you see is the slice of the etalon interference pattern that is illuminated by the line. Adjust the alignment of the etalon, using the controls on the spectrometer, so that when a line is centered in the field of view, it lies centred across a diameter of the pattern, see figure 4. It is the spacing of the segments, and in particular what happens at the center of the pattern, that gives a precision measurement of the wavelength.
Etalon theory and the exact fractions method

A Fabry-Perot etalon consists of two optically flat partially reflecting mirrors that are maintained parallel to each other. Circular interference fringes are produced when it is illuminated with parallel light. In figure 5, $t$ is the mirror spacing and $\theta$ is the angle of observation through the telescope, thus a constant $\theta$ represents a circle in the observed pattern (which may or may not coincide with a bright fringe). $\theta = 0$ is the centre of the pattern.

The constructive interference necessary for a bright fringe occurs when the rays 1, 2, 3... allowed through after increasing numbers of partial reflections are all in phase, modulo $2\pi$. i.e. the extra path length added by each pair of reflections is a whole number of wavelengths. Show that the extra path length $BCD$ is equal to $2t\cos \theta$. The dotted construction lines may help. Notice that this result means that the path...
difference decreases as $\theta$ increases. The value of $2t$, the path difference at normal incidence, will not generally be a whole number of wavelengths, so we will write $2t = \lambda (k + f)$, where $\lambda$ is the wavelength, $k$ is an integer and $f$ is a fraction (between 0 and 1). $k$ and $f$ are not yet known, because $2t$ is only known accurately once we have analysed the data for some known wavelengths. Clearly, $k$ and $f$ will be different for each wavelength.

In the focal plane of the telescope, we see a series of bright rings of radii $r_1$, $r_2$, $r_3$... formed by light passing through the etalon at angles $\theta_1, \theta_2, \theta_3...$, where $\theta_i = r_i/s$ ($s$ is the focal distance and $i$ is the ring number), see figure 6.

![Figure 6: Fringe geometry (lenses omitted)](image)

Only if $f$ happened to be 0 would there be a bright fringe at $r = 0$. The first fringe ($i = 1$) is normally found when $2t \cos \theta_1 = k\lambda$, the second when $2t \cos \theta_2 = (k - 1)\lambda$, and in general, fringe $i$ is where $2t \cos \theta_i = (k - i + 1)\lambda$. The angle $\theta$ is always small, so we can approximate $\cos \theta \approx 1 - \frac{1}{2} \theta^2$ and substitute $\theta_i = r_i/s$, giving

$$r_i^2 = \frac{s^2 \lambda}{t} i + s^2 \left( 2 - \frac{\lambda (k + 1)}{t} \right)$$  (1)

which means that a plot of $r_i^2$ versus $i$ should be a straight line having an intercept on the $i$ axis at $k + 1 - 2t/\lambda$. Since we previously defined $k + f = 2t/\lambda$, the intercept is also equal to $1 - f$. 

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In our calibration procedure, we will use this relationship to measure the value of $f$ for each known wavelength $\lambda$. The corresponding values of $k$ will remain unknown. However, if we guess a value for $t$, we can work out $k = \frac{2t}{\lambda} - f$. The guess can be right only if $k$ comes out as an integer (to within experimental error). When we find a single value of $t$ that makes each of the $k$ values an integer for all of the wavelengths we use, we have found the correct $t$. Combining the results for a few known wavelengths, there should be only one reasonable value of $t$. This is known as an *exact fractions* method: we don’t initially know the values of $k$, but we know they must be integers.

Once we know $t$, it is straightforward to find the wavelength of an unknown spectral line from its fringe positions.

**Fringe measurements**

Centre a spectral line in the field of view, remove the spectrometer eyepiece, and replace it with the TV camera. The GUVCVIEW application will show the camera image. Adjust the controls of the spectrometer and camera to give a good view of the fringes. Save a snapshot of the image, and drag it into IMAGEJ, where you can select a rectangular region and get a profile of the etalon fringes along a spectral line (Analyze | Plot profile, or ctrl-k).

Use the profile to check that the fringes are as sharp as possible, then carefully read the fringe positions from the profile. The position measurements are in camera sensor pixels.

It is important to locate the centres of the fringes accurately, and to have a way of finding the center of the pattern (i.e. the pixel number for $r = 0$). You should consider several methods of measuring the ring radii. For example, you could measure diameters and divide the results by 2, locate $r = 0$ by eye, or treat the $r = 0$ value as an unknown to be determined at the same time as equation 1 is fitted. You should also estimate the errors on your measurements.

Repeat the fringe measurements for each of the known cadmium lines, and for the less well known line near 441 nm.

**Analysis**

Use equation 1 to determine $f$ for each of the known lines. You could calculate $r^2$ and then perform a straight line fit versus $i$, or fit the nonlinear function itself. The QTI-PLOT data analysis application on the lab PC will do either, or you can try any other
techniques you know. If doing a nonlinear fit, you might try fitting \( r = r_0 + \sqrt{M t + C} \), where the intercept \( C/M \) is the required intercept, and \( r_0 \), the central position, is another unknown.

The etalon is marked with its nominal thickness \( t \). This is not accurate enough for wavelength determination, but it gives a starting value for your guessed values of \( t \). Using a spreadsheet or similar technique, try perhaps a few hundred values of \( t \) within 10 \( \mu \)m of the nominal value, evaluating \( k = \left( \frac{2t}{\lambda} \right) - f \) for all of the known wavelengths. Look for the value of \( t \) that makes all of the \( k \) values close to integers at the same time. When you have found this \( t \), use it to find the unknown wavelength.

Since the purpose of this experiment is to make a precision measurement, you should show clearly in your lab book how you have handled the experimental errors, and calculated the probable error in your result, as well as recording how you acquired and analysed the data.

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